

Charged pion polarizability in the nonlocal quark model of Nambu–Jona-Lasinio type

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Abstract

The polarizability of a charged pion is estimated in the framework of the nonlocal chiral quark model of the Nambu–Jona-Lasinio type. Nonlocality is described by quark form factors of the Gaussian type. It is shown that the polarizability in this model is very sensitive to the form of nonlocality and choice of the model parameters.

Recently, the interest to direct determination of the pion polarizability in Primakoff scattering has been renewed. Now the new experiment at CERN is prepared by the collaboration COMPASS where it is expected to obtain the statistics factor 6000 higher than in the previous experiment performed by the IHEP and JINR group in Protvino [1] at the beginning of the 80s. However, the technical possibilities of this experiment allow one to obtain only very rough estimations of the pion polarizability with large error bars

$$\alpha_\pi = -\beta_\pi = 8.54 \pm 1.76 \pm 1.51, \quad (1)$$

where α_π and β_π are electric and magnetic polarizabilities of the charged pion. Hereafter we express the polarizability in the units of 10^{-42}cm^3 . The new experiment gives an opportunity to obtain a value of the pion polarizability with good accuracy [2].

What concerns the theoretical aspect of this problem for the past years, there are a lot of estimations of this quantity made by many authors in different theoretical models, in particular, one of the authors of this article (MKV) together with V. N. Pervushin obtained estimations of the pion polarizability in the framework of the nonlinear chiral model in 1975 [3]. After that, analogous calculations were performed by him in the quark linear sigma model of Nambu–Jona-Lasinio(NJL) type [4, 5]. The estimation obtained in the above-mentioned works corresponds to the experimental result (1).

In this short note, we want to return to this question and estimate the pion polarizability in the nonlocal model of the NJL type [6], where nonlocality described by quark form factors of the Gaussian type.

As it is shown in [5], in the local NJL model the main contribution to the pion polarizability in the leading order of $1/N_c$ expansion stems from two types of diagrams: the diagrams with light intermediate σ -meson, and the box diagrams (see fig.1). The contribution from the diagrams with other resonances (heavier scalars, vectors and axial-vectors) is smaller than 3% and can be neglected in the present consideration.

The contribution of the box diagrams to the structure part of the Compton effect takes the form

$$A_{(a)}^{\mu\nu} = \frac{\alpha}{9\pi f_\pi^2} (g^{\mu\nu} (q_1 \cdot q_2) - q_2^\mu q_1^\nu),$$

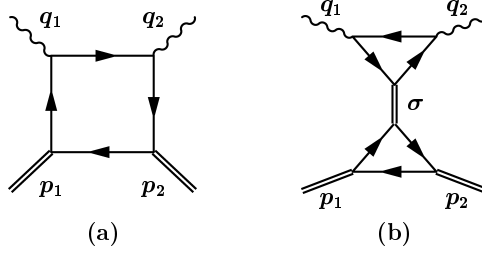


Figure 1: Diagrams describing charge pion polarizability: a) box diagrams; b) σ pole diagrams.

where q_1, q_2 are the momenta of the incoming(outgoing) photons, $\alpha = 1/137$ is the fine structure constant, $f_\pi = 93$ MeV is the weak pion decay constant. The contribution of the σ pole diagram can be factorized into three parts

$$A_{(b)}^{\mu\nu} = A_{\sigma\pi\pi} D_\sigma A_{\sigma\gamma\gamma}^{\mu\nu},$$

where the $\sigma\pi\pi$ vertex is $A_{\sigma\pi\pi} = 4mgZ = 4m^2 Z^{1/2}/f_\pi$, the σ meson propagator is $D_\sigma \approx 1/M_\sigma^2$ and the $\sigma\gamma\gamma$ vertex is [7]

$$A_{\sigma\gamma\gamma}^{\mu\nu} = (g^{\mu\nu}(q_1 \cdot q_2) - q_2^\mu q_1^\nu) \frac{10\alpha Z^{-1/2}}{9\pi f_\pi},$$

here $m = 280$ MeV is the constituent quark mass after taking into account the $\pi - a_1$ mixing [5], $g = mZ^{-1/2}/f_\pi$ is the σ meson coupling constant, factor Z is $Z = (1 - 6m^2/M_{a_1}^2)^{-1} \approx 1.4$ ($M_{a_1} = 1.26$ GeV is the mass of the a_1 meson), M_σ is the σ meson mass $M_\sigma = \sqrt{4m^2 + M_\pi^2} \approx 2m$ and M_π is the pion mass. As a result, the pion polarizability takes the form

$$\alpha_\pi = -\beta_\pi = \frac{5\alpha}{9\pi M_\pi f_\pi^2} \left(1 - \frac{1}{10}\right) = 7.3, \quad (2)$$

One can see that the polarizability does not depend on the model parameters.

Let us emphasize that the contribution of the box diagrams makes up only 10 % of the contribution of the σ pole diagrams¹.

Our calculation shows that in the nonlocal model the choice of the form-factor $f(p)$ and the model parameters have more influence on the pion polarizability than taking into account the box diagrams. Therefore, in this note we restrict ourselves only with the estimations of the σ pole diagrams contribution.

In the nonlocal, the model quark mass depends on momentum. In [6], the following representation for the quark propagator is proposed

$$\frac{m^2(p)}{m^2(p) + p^2} = \exp(-p^2/\Lambda^2), \quad (3)$$

¹It is worth noting that a similar situation with relative contributions from the σ pole and box diagrams takes place in nonlocal model (see, e.g., [8]).

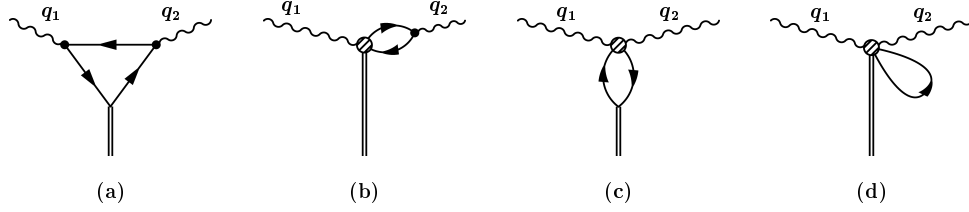


Figure 2: Diagrams describing vertex $\sigma\gamma\gamma$.

where Λ is the parameter of nonlocality. As a result, the pole part of the quark propagator has no singularities on the whole real axis, what leads to the quark confinement. The quark mass function takes the form [6]:

$$m^2(p) = \left(\frac{p^2}{\exp(p^2/\Lambda^2) - 1} \right). \quad (4)$$

From this equation one can see that $m(0) = \Lambda$. Then, from the condition that the weak pion decay constant equals 93 MeV, it follows that $m(0) = \Lambda = 340$ MeV. For the $\sigma\pi\pi$ vertex and the σ meson mass we have $A_{\sigma\pi\pi} = 1.57$ GeV, $M_\sigma = 420$ MeV [6]. The vertex $\sigma\gamma\gamma$ is more complicated. We can express it as

$$A_{\sigma\gamma\gamma}^{\mu\nu} = \alpha C_{\sigma\gamma\gamma} (g^{\mu\nu} (q_1 \cdot q_2) - q_2^\mu q_1^\nu).$$

Due to the $\mathcal{P}\exp$ factor in the action [6, 9], there are additional nonlocal photon vertices. The reason for appearing of these vertices is the momentum dependence of the quark mass and the meson-quark vertices. The technique of obtaining of these vertices can be found in [9, 10]. As a result, there are four types of diagrams describing the process $\sigma\gamma\gamma$, fig.2, only sum of them is gauge invariant. As a result, we have²

$$C_{\sigma\gamma\gamma} = 0.86 \text{ GeV}^{-1},$$

and pion polarizability is

$$\alpha_\pi = 1.5.$$

Let us notice that the calculations of the pion polarizability in a similar nonlocal model of the NJL type with quark form-factors of the Gaussian type is performed in [11] by using the chiral sum rule method. Those form-factors lead to the following momenta dependence of quark mass function

$$m(p) = m_0 \exp(-2p^2/\Lambda^2).$$

In contrast to the previous model, here the only one condition is used $f_\pi = 93$ MeV for fixing two main model parameters m_0 , Λ . This keeps some freedom in choosing model parameters. Particularly, in [11] the model parameters $m_0 = 300$ MeV, $\Lambda = 1.085$ GeV are used. As a result, the values of the pion polarizability obtained by the sum rule method

²It is easy to see that $A_{\sigma\pi\pi}$, $C_{\sigma\gamma\gamma}$ noticeably decrease in comparison with the local NJL model (see also table 1). We would like to emphasize that similar situation takes place in the other nonlocal model with form factors of the Gaussian type [12].

model	$m(0)$ MeV	Λ MeV	M_σ MeV	$A_{\sigma\pi\pi}$ GeV	$C_{\sigma\gamma\gamma}$ GeV ⁻¹	α_π sigma pole 10 ⁻⁴² cm ³	α_π sum rules 10 ⁻⁴² cm ³
[6]	340	340	420	1.57	0.86	1.5	2.0
[11]	340	920	421	1.57	1.16	2.0	2.7
[11]	300	1085	398	1.47	1.75	3.3	3.6
[11]	280	1187	384	1.39	2.1	3.9	5.0
[5]	280	1250	577	4	3.2	8.1	

Table 1: Theoretical values of pion polarizability.

is $\alpha_\pi = 3.6$. On the other hand, we can carry out calculations of the pion polarizability using the σ pole diagrams. We have

$$\begin{aligned} A_{\sigma\pi\pi} &= 1.47 \text{ GeV}, \\ M_\sigma &= 398 \text{ MeV}, \\ C_{\sigma\gamma\gamma} &= 1.75 \text{ GeV}^{-1}, \end{aligned} \tag{5}$$

and, as a result, we obtain $\alpha_\pi = 3.3$. It worth noting that if m_0 is equal to 340 MeV, as in model [6] (see eq.(4)), the model quantities equal

$$\begin{aligned} \Lambda &= 0.92 \text{ GeV} \\ A_{\sigma\pi\pi} &= 1.57 \text{ GeV}, \\ M_\sigma &= 421 \text{ MeV}, \\ C_{\sigma\gamma\gamma} &= 1.16 \text{ GeV}^{-1}, \end{aligned} \tag{6}$$

the polarizability of the pion is $\alpha_\pi = 2.0$ (sum rule estimation is $\alpha_\pi = 2.7$). If one takes $m_0 = 280$ MeV, as in the local NJL model, one can obtain³

$$\begin{aligned} \Lambda &= 1.187 \text{ GeV}, \\ A_{\sigma\pi\pi} &= 1.39 \text{ GeV}, \\ M_\sigma &= 384 \text{ MeV}, \\ C_{\sigma\gamma\gamma} &= 2.1 \text{ GeV}^{-1}, \\ \alpha_\pi &= 3.9 (\alpha_\pi = 5.0 \text{ from the sum rules}). \end{aligned} \tag{7}$$

As we can see, in the nonlocal model there is a strong dependence of the pion polarizability on the form of nonlocality. We summarize theoretical results in the table 1.

The experimental values of pion polarizability are given in the table 2.

To conclude, we consider the estimation of the pion polarizability in the framework of the local and nonlocal models of the NJL type. It is shown that the pion polarizability in the nonlocal model is noticeably smaller than in the local NJL model, and is very sensitive to the form of nonlocality and choice of the model parameters. Planned experiments must give more accurate values of the pion polarizability. We hope that these data allow us to chose more realistic version of the nonlocal model.

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³Note that in the local NJL model, Λ equals 1.25 GeV [5].

$\alpha_{\pi}^{\text{exp}}$	experiment
$8.54 \pm 1.76 \pm 1.51$	[1]
3.3 ± 0.6	[13]
3.31 ± 0.45	[14]
3.4 ± 1.11	[15]

Table 2: Experimental values of the pion polarizability: [1] measurement via Primakoff scattering; [13] deduced from the processes $\gamma\gamma \rightarrow \pi\pi$; [14], [15] obtained from the sum rules for vector and axial-vector correlation functions in τ decays.

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